ON THE DESIGN OF MINIMUM BER LINEAR MIMO TRANSCEIVERS WITH PERFECT OR PARTIAL CHANNEL STATE INFORMATION

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ABSTRACT

Linear MIMO transceivers (composed of a linear precoder at the transmitter and a linear equalizer at the receiver) are a low-complexity approach to optimize the spectral efficiency and/or the reliability of the communication, when channel state information is available at both sides of the link. In this paper we focus on the minimum BER linear transceiver design. Based on the observation that the common practice of fixing a priori the number of transmitted data symbols per channel use inherently limits the diversity gain of the system, we derive a minimum BER linear precoding scheme that achieves the full diversity of the MIMO channel. Finally, we adapt the proposed design to the limited feedback case, i.e. when the optimum transmit matrix is selected from a codebook at the receiver. Numerical simulations show that both schemes outperform the classical minimum BER designs exiting in the literature.

Index Terms— minimum BER design, linear MIMO transceiver, perfect CSI, partial CSI, limited feedback

1. INTRODUCTION

The increasing demand of higher data rates, specifically in wireless communications, has motivated interest in the design and analysis of multiple-input multiple-output (MIMO) systems. Specifically, when channel state information (CSI) is accessible simultaneously at the transmitter (CSI-T) and at the receiver (CSI-R), the MIMO system can be adapted to each channel realization to maximize the spectral efficiency and/or reliability of the communication. Theoretically, the optimal transmission is given by a Gaussian signaling with a waterfilling power profile over the channel eigenmodes [1]. From a more practical standpoint, however, the ideal Gaussian codes are substituted with practical constellations (such as QAM constellations) and coding schemes. To simplify the study of such a system, it is customary to divide it into an uncoded part, which transmits symbols drawn from some constellations, and a coded part that builds upon the uncoded system. Although the ultimate system performance depends on the combination of both parts, it is convenient to consider the uncoded and coded parts independently to simplify the analysis and design.

This paper focuses on the uncoded part of the system and, specifically, on the employment of linear transceivers (composed of a linear precoder at the transmitter and a linear equalizer at the receiver). Under the perfect CSI assumption, the design of linear transceivers has been extensively treated in the literature according to a variety of criteria [2, 3, 4, 5]. This paper concentrates only on the design that minimizes bit error probability (BER), since it measures the ultimate performance of an uncoded digital communication system.

First, we present the minimum BER design obtained assuming that the number of data symbols per channel use (denoted by K) has been previously chosen and equal symbol constellations are employed. The classical minimum BER linear transceiver with fixed and equal constellations, derived simultaneously in [4] and [5], transmits a rotated version of the data symbols through the K strongest channel eigenmodes in a waterfilling fashion. The analytical performance characterization in [6] shows that the diversity gain of this scheme is given by $(n_T-K+1)(n_R-K+1)$, where n_T is the number of transmit and n_R the number of receive antennas. This suggests that the average BER could be improved by introducing the number of active substreams in the design criterion. This extra degree of freedom is utilized in the proposed scheme by fixing the global rate but allowing the use of an adaptive symbol constellation to compensate for the change in the number of active substreams. Besides, and for the sake of simplicity, the constellation is assumed to be the same in all substreams. This scheme is named multimode minimum BER linear transceiver, and, in contrast to the classical minimum BER design, does fully exploit the diversity gain $n_R n_T$ of the MIMO channel. Note that a more general setup would also adapt the individual modulations without the equal constellations constraint. However, even the minimum BER linear transceiver with fixed and unequal constellations can not be optimally obtained in closed form [7], and the proposed multimode minimum BER scheme already achieves the full diversity of the channel with low complexity.

Due to the random time-varying nature of wireless channels it is difficult to obtain perfect CSI-T in a realistic scenario. In some cases the transmit matrix can be inferred from the receive channel through reciprocity, but more often the CSI is reported to the transmitter using a limited-rate feedback channel. In this situation, the optimum precoder is selected at the receiver from an offline constructed codebook and sent to the transmitter using the feedback channel [8, 9, 10]. Based on the multimode minimum BER scheme derived under the perfect CSI-T assumption, we finally design the limited feedback multimode minimum BER linear transceiver, which also fully exploits the diversity gain of the MIMO channel. Thus, in this paper we propose two minimum BER and full diversity schemes for the case of having perfect or partial CSI-T.

The rest of the paper is organized as follows. Section 2 is devoted to introducing the signal model and presenting the average BER performance measure. The minimum BER linear transceiver design is addressed in Section 3 under the perfect CSI-T assumption and in Section 4 under the partial CSI-T assumption. Finally, we summarize the main contribution of the paper in the last section.

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2. SYSTEM MODEL AND PERFORMANCE MEASURE

The signal model corresponding to a transmission through a general MIMO channel with n_T transmit and n_R receive antennas is

$$\mathbf{y} = \sqrt{\frac{\mathsf{snr}}{K}}\mathbf{H}\mathbf{x} + \mathbf{w} \tag{1}$$

where $\mathbf{x} \in \mathbb{C}^{n_T \times 1}$ is the transmitted vector, $\mathbf{H} \in \mathbb{C}^{n_R \times n_T}$ is the channel matrix, $\mathbf{y} \in \mathbb{C}^{n_R \times 1}$ is the received vector, and $\mathbf{w} \in \mathbb{C}^{n_R \times 1}$ is a spatially white zero-mean circularly symmetric complex Gaussian noise vector normalized so that $\mathrm{E}\{\mathbf{w}\mathbf{w}^\dagger\} = \mathbf{I}_{n_R}$, and snr is the average SNR per receive antenna. The channel matrix $\mathbf{H}$ contains the complex path gains $[\mathbf{H}]_{ij}$ between every transmit and receive antenna pair. We adopt an uncorrelated Rayleigh flat-fading channel model and, consequently, these coefficients are independent complex Gaussian random variables with zero mean and unit variance.

Suppose that the MIMO communication system is equipped with a linear transceiver (see Fig. 1), then the transmitted vector is given by

$$\mathbf{x} = \mathbf{B}_K \mathbf{s}_K. \tag{2}$$

where $\mathbf{B}_K \in \mathbb{C}^{n_T \times K}$ is the transmit matrix (precoder) and the data vector $\mathbf{s}_K \in \mathbb{C}^{K \times 1}$ gathers the $K \leq \min\{n_T, n_R\}$ data symbols to be transmitted (zero mean, with unit energy and uncorrelated, i.e. $\mathrm{E}\{\mathbf{s}_K \mathbf{s}_K^\dagger\} = \mathbf{I}_K$). We consider a fixed-rate data transmission and, hence, each data symbol $s_{k,K}$ is drawn from a fixed M_K-dimensional constellation such that the total transmission rate $R = K \log_2 M_K$[1] is fixed for all channel realizations and all possible values of K. The transmitted power is constrained such that (see [11, 12] for details)

$$\lambda_{\max}\left\{\mathrm{E}\left\{\mathbf{x}\mathbf{x}^\dagger\right\}\right\} = \lambda_{\max}\left\{\mathbf{B}_K \mathbf{B}_K^\dagger\right\} \leq 1. \tag{3}$$

The estimated data vector at the receiver is given by

$$\hat{\mathbf{s}}_K = \mathbf{A}_K^\dagger \mathbf{y} = \mathbf{A}_K^\dagger \left(\sqrt{\frac{\mathsf{snr}}{K}}\mathbf{H}\mathbf{B}_K \mathbf{s}_K + \mathbf{w}\right). \tag{4}$$

where $\mathbf{A}_K^\dagger \in \mathbb{C}^{K \times n_R}$ is the receive matrix (equalizer).

The ultimate performance of MIMO linear transceivers is measured by the BER averaged over the K data symbols to be transmitted:

$$\mathrm{BER}_K(\mathsf{snr}) = \frac{1}{K}\sum_{k=1}^{K} \mathrm{BER}_{k,K}(\rho_{k,K}) \tag{5}$$

where $\rho_{k,K}$ is the instantaneous SNR and $\mathrm{BER}_{k,K}(\rho_{k,K})$ is the instantaneous BER of the k^{th} substream (out of K active):

$$\mathrm{BER}_{k,K}(\rho_{k,K}) = \frac{\alpha_K}{\log_2 M_K}\mathcal{Q}\left(\sqrt{\beta_K \rho_{k,K}}\right) \tag{6}$$

where α_K and β_K are parameters of the M_K-dimensional constellation (see e.g. [13]).

3. MINIMUM BER LINEAR TRANSCEIVER WITH PERFECT CSI-T

3.1. Classical Linear Transceiver Design and Performance

Assuming perfect CSI at the transmitter and at the receiver, the design of the optimal linear MIMO transceiver when fixing K beforehand is formulated in [4] as the minimization of a certain cost function of mean-square errors (MSEs), since the BER can be easily related to the MSE. Specifically, [4] shows that the optimum receive

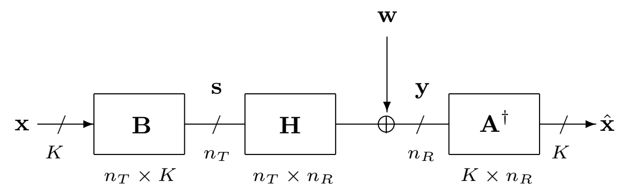

Fig. 1. Linear MIMO transceivers system model.

matrix $\mathbf{A}_K$, for a given transmit matrix $\mathbf{B}_K$, is given by the Wiener filter solution:

$$\mathbf{A}_K = \sqrt{\frac{K}{\mathsf{snr}}}\left(\mathbf{H}\mathbf{B}_K\mathbf{B}_K^\dagger\mathbf{H}^\dagger + \frac{K}{\mathsf{snr}}\mathbf{I}_{n_R}\right)^{-1}\mathbf{H}\mathbf{B}_K \tag{7}$$

independently of the design cost function. Under the minimum BER design criterion, the precoder matrix $\mathbf{B}_K$ is obtained as

$$\mathbf{B}_K = \arg\min_{\mathbf{B}_K} \mathrm{BER}_K(\mathsf{snr}) \tag{8}$$

subject to the power constraint in (3). The optimum transmit matrix $\mathbf{B}_K$ is given by [4, 5]

$$\mathbf{B}_K = \mathbf{U}_K\sqrt{\mathbf{P}_K}\mathbf{Q}_K \tag{9}$$

where $\mathbf{U}_K \in \mathbb{C}^{n_T \times K}$ has as columns the eigenvectors of $\mathbf{H}^\dagger\mathbf{H}$ corresponding to the K largest nonzero eigenvalues $\lambda_1 \geq \cdots \geq \lambda_K$, $\mathbf{Q}_K \in \mathbb{C}^{K \times K}$ is a unitary matrix such that $\left(\mathbf{I}_K + \mathbf{B}_K^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{B}_K\right)^{-1}$ has identical diagonal elements (see [4] for details), and $\mathbf{P}_K \in \mathbb{C}^{K \times K}$ is a diagonal matrix containing the power allocation. The peak-power constraint in (3) does not modify the eigenvector structure of the optimal precoder given in [4, 5] but it has a significant effect on the power allocation among the precoder singular values (diagonal elements of $\sqrt{\mathbf{P}_K}$). In contrast to the most common sum-power constraint, when using a maximum eigenvalue constraint as in (3), unequal power pouring only reduces the total transmitted power. The optimum precoder should have the largest possible singular values and, hence, the optimum power allocation is given by

$$p_{k,K} = [\mathbf{P}_K]_{kk} = 1 \qquad k = 1, \ldots, K. \tag{10}$$

which implies that the optimum precoder in a unitary matrix.

Given the optimum transmit matrix in (9) and the optimum receive matrix in (7), the communication process is diagonalized up to a specific rotation $\mathbf{Q}_K$ that forces all data symbols to have the same MSE [4]

$$\mathsf{mse}_K \triangleq \mathsf{mse}_{k,K} = \frac{1}{K}\sum_{i=1}^{K}\left(\frac{\mathsf{snr}}{K}\lambda_i + 1\right)^{-1} \tag{11}$$

and, hence, the same instantaneous SNR [4]

$$\rho_K \triangleq \rho_{k,K} = \mathsf{mse}_K^{-1} - 1 = \left(\frac{1}{K}\sum_{i=1}^{K}\left(\frac{\mathsf{snr}}{K}\lambda_i + 1\right)^{-1}\right)^{-1} - 1. \tag{12}$$

Thus, the classical minimum BER design transmits a rotated version of the K data symbols through the K strongest channel eigenmodes, so that all data symbols experience the same BER performance. The instantaneous BER averaged over the K data symbols defined in (5) is then given by

$$\mathrm{BER}_K(\mathsf{snr}) = \frac{\alpha_K}{\log_2 M_K}\mathcal{Q}\left(\sqrt{\beta_K \rho_K}\right). \tag{13}$$

[1] Note that K and $\log_2 M_K$ have to be integers.

Now, taking into account different channel realizations, the average BER is obtained as

$$\begin{aligned}\overline{\mathrm{BER}}_K(\mathsf{snr}) &= \mathrm{E}\left\{\mathrm{BER}_K(\mathsf{snr})\right\} \\ &= \frac{\alpha_K}{\log_2 M_K}\int_0^{\infty}\mathcal{Q}\left(\sqrt{\beta_K\rho}\right)f_{\rho_K}(\rho)d\rho \quad (14)\end{aligned}$$

where $f_{\rho_K}(\rho)$ is the pdf of the instantaneous SNR, ρ_K, given in (12). Under the Rayleigh fading assumption, ρ_K is a function of the K strongest eigenvalues of the Wishart distributed channel matrix $\mathbf{H}^{\dagger}\mathbf{H}$. Since tractable close-form expressions for the marginal pdfs of the ordered eigenvalues have not been derived, the average BER in (14) cannot be analytically computed. In [6], the high-SNR performance of the classical minimum BER linear transceiver has been characterized in terms of two key parameters (the array gain and the diversity gain). The diversity gain represents the slope of the BER curve at high SNR and the array gain (also known as coding gain) determines the horizontal shift of the BER curve. In particular, the diversity gain is given by [6]

$$G_{d,K} = (n_T - K + 1)(n_R - K + 1). \tag{15}$$

which is the diversity gain of the K^{th} substream [13]. Hence, the performance of this scheme is limited by the inherent performance of K^{th} channel eigenmode. This reveals that the average BER can be improved by introducing K into the design criterion, as proposed in the following section.

3.2. Multimode Linear Transceiver Design and Performance

In contrast to classical linear precoding, where the number of data symbols to be transmitted per channel use K is fixed, in the multimode minimum BER linear transceiver, K and the M_K-dimensional constellations are adapted to the instantaneous channel conditions by allowing K to vary between 1 and $n = \min\{n_T, n_R\}$ keeping the total transmission rate $R = K\log_2 M_K$ fixed. Usually, only a subset $\mathcal{K}$ of all n possible values of K is supported, since the number of bits per symbol R/K has to be an integer.

The linear precoder $\mathbf{B}_K$ and K are designed to minimize the BER averaged over the data symbols to be transmitted for all supported values of K:

$$\{K, \mathbf{B}_K\} = \arg\min_{K,\,\mathbf{B}_K}\mathrm{BER}_K(\mathsf{snr}) \tag{16}$$

where $\mathrm{BER}_K(\mathsf{snr})$ is defined in (5), $K \in \mathcal{K}$, and $\mathbf{B}_K$ has to satisfy the power-constraint in (3). The optimum linear precoder $\mathbf{B}_K$ for a given K has been presented and analyzed in Section 3.1. Given the instantaneous BER expression in (13), K should be optimally selected as

$$K = \arg\min_{K\in\mathcal{K}}\frac{\alpha_K}{\log_2 M_K}\mathcal{Q}\left(\sqrt{\beta_K\rho_K}\right) \tag{17}$$

or, neglecting the contribution of $\alpha_K/\log_2 M_K$ (since it is not in the argument of the $\mathcal{Q}$-function), as

$$K = \arg\max_{K\in\mathcal{K}}\beta_K\rho_K \tag{18}$$

where ρ_K is given in (12). Once the optimum mode K has been selected, the optimum precoder is constructed as in (9).

The average BER of the multimode minimum BER scheme, denoted by $\overline{\mathrm{BER}}(\mathsf{snr})$, can be upper-bounded using Jensen's inequality as

$$\overline{\mathrm{BER}}(\mathsf{snr}) = \mathrm{E}\left\{\min_{K\in\mathcal{K}}\mathrm{BER}_K(\mathsf{snr})\right\} \leq \min_{K\in\mathcal{K}}\overline{\mathrm{BER}}_K(\mathsf{snr}). \tag{19}$$

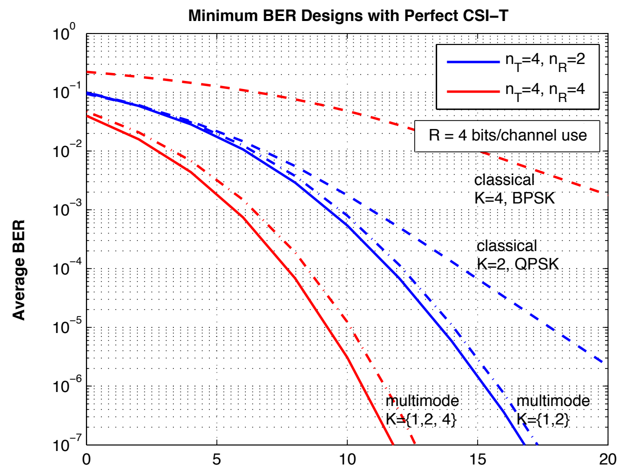

Fig. 2. Simulated average BER of the classical minimum BER linear transceiver (dashed), the multimode minimum BER linear transceiver (solid), and the multimode precoder (dash-doted).

This shows that the multimode linear transceiver outperforms the classical linear transceiver for all supported modes. Then, assuming that mode $K = 1$ is contained in $\mathcal{K}$, it follows that

$$\overline{\mathrm{BER}}(\mathsf{snr}) \leq \overline{\mathrm{BER}}_1(\mathsf{snr}) \tag{20}$$

where $\overline{\mathrm{BER}}_1(\mathsf{snr})$ denotes the average BER of the classical linear transceiver design when using only one substream, i.e a beamforming strategy. Beamforming is known to achieve the full diversity of the channel [14], $n_T n_R$, and, hence, the diversity gain of the multimode minimum BER linear transceiver is also given by

$$G_d = n_T n_R \tag{21}$$

whenever $K = 1$ is a supported mode.

3.3. Numerical Results

In Fig. 2 we show the average BER performance of the multimode minimum BER linear transceiver (solid) in a Rayleigh flat-fading channel. In addition, we have included the classical minimum BER design (dashed) for $K = \min\{n_T, n_R\}$ and a suboptimum scheme (dash-dotted lines) that also adapts K jointly with the precoder and achieves full diversity. This technique is called multimode precoding and was proposed in [10] in the context of limited feedback linear precoding. As expected, the proposed design outperforms the classical minimum BER linear transceiver and the suboptimum scheme of [10]. Although the linear precoder proposed in [10] also includes the mode selection procedure, it does not perform the rotation to ensure equal BER on all substreams and the proposed optimum mode selection. This is exactly the reason for the average BER gap that can be seen in Fig. 2.

4. MINIMUM BER LINEAR TRANSCEIVER WITH PARTIAL CSI-T

4.1. Limited Feedback Approach

When perfect CSI-T is available, the precoder $\mathbf{B}$ is designed at the transmitter as a function of the instantaneous channel conditions to minimize some performance measure (as in Section 3). However, when the channel cannot be estimated at the transmitter and the feedback channel is capacity-constrained ($b = \log_2 N$ bits per channel use), the transmit matrix $\mathbf{B}$ is chosen at the receiver by means of a selection function $f : \mathbb{C}^{n_R \times n_T} \rightarrow \mathcal{B} = \{\mathbf{B}^{(1)}, \mathbf{B}^{(2)}, ..., \mathbf{B}^{(N)}\}$ from a codebook of N elements and then conveyed to the transmitter using the feedback channel. The precoding codebook $\mathcal{B}$ is designed offline using the following procedure: (i) a channel-dependent distortion measure is proposed and (ii) the codebook is obtained as the set of N matrices that minimizes the distortion averaged over all possible channel states. This distortion function must differ, however, from the distortion functions commonly used in vector quantization such as the MSE because we are interested in improving the system performance rather than improving the quality of the estimated precoder at the transmitter [15].

4.2. Limited Feedback Multimode Linear Transceiver

In this section, we present the limited feedback version of the minimum BER multimode linear transceiver introduced in Section 3.2. Under the limited feedback assumption, the system uses a different codebook $\mathcal{B}_K$ for each one of the supported modes $K \in \mathcal{K}$. Each codebook contains N_K precoders of dimensions $n_T \times K$, such that

$$\sum_{K \in \mathcal{K}} N_K = N = 2^b \qquad (22)$$

where b is the number of feedback bits. Then, for a given channel realization, the receiver selects the best transmit matrix over all precoders contained in the $|\mathcal{K}|$ codebooks and feeds the corresponding index back to the receiver.

Let us start with the design of the precoder selection function to obtain the optimum precoder from individual codebooks $\{\mathcal{B}_K\}_{K \in \mathcal{K}}$. Recalling the result presented in (9) and (10), the optimum precoder for a given K is

$$\mathbf{B}_K = \tilde{\mathbf{B}}_K \mathbf{Q}_K = \mathbf{U}_K \mathbf{Q}_K. \qquad (23)$$

Noting that $\mathbf{Q}_K$ is a channel non-dependent rotation matrix that assures equal performance on the K substreams [4], only the contribution of the unitary matrix $\tilde{\mathbf{B}}_K = \mathbf{U}_K$, which contains the K eigenvectors associated with the K strongest eigenvalues of $\mathbf{H}^\dagger \mathbf{H}$, has to be considered in the design of the codebook $\mathcal{B}_K$.

The BER performance of the minimum BER multimode design is characterized by the SNR or the MSE performance of the transmitted substreams (see Section 3). Thus, the optimum precoder is given by

$$\mathbf{B}_{K,\mathrm{opt}} = \tilde{\mathbf{B}}_{K,\mathrm{opt}} \mathbf{Q}_K \qquad (24)$$

where $\tilde{\mathbf{B}}_{K,\mathrm{opt}}$ is selected from the codebook $\mathcal{B}_K$ as

$$\tilde{\mathbf{B}}_{K,\mathrm{opt}} = \arg \max_{\tilde{\mathbf{B}}_K \in \mathcal{B}_K} \beta_K \rho_K \qquad (25)$$

where

$$\rho_K = \left(\frac{1}{K} \mathrm{Tr} \left\{ \left(\frac{\mathsf{snr}}{K} \tilde{\mathbf{B}}_K^\dagger \mathbf{H}^\dagger \mathbf{H} \tilde{\mathbf{B}}_K + \mathbf{I}_K \right)^{-1} \right\} \right)^{-1} - 1. \qquad (26)$$

Once the selection function in (25) has been proposed, we can start with the codebook design problem. The codebook $\mathcal{B}_K$ is optimally constructed as the set of N_K precoders ($n_T \times K$ unitary matrices) that minimize a predefined distorsion measure averaged over all possible channel realizations. This distortion measure has to compare the BER performance achieved with the designed codebook with respect to the optimum BER performance (when perfect CSI-T is available). However, any distortion function directly based on the selection function in (25) is SNR-dependent and, thus, it results in a different design criterion for each SNR value which is impractical, due to the large SNR operation range of wireless communication systems. Nevertheless, it has been shown in [16], that the average BER performance of limited feedback linear transceivers is not very sensitive to the codebook design criterion and that performance advantages may fundamentally come from the suitable choice of the precoder selection function. Hence, we can use an approximate SNR-independent distortion function to design the codebook and use the selection function in (25) to obtain the optimum precoder without expecting significant performance losses.

The MSE of limited feedback multimode design can be upper-bounded as

$$\begin{aligned} \mathsf{mse}_K &= \frac{1}{K} \sum_{i=1}^{K} \left(\frac{\mathsf{snr}}{K} \lambda_k \{ \tilde{\mathbf{B}}_K^\dagger \mathbf{H}^\dagger \mathbf{H} \tilde{\mathbf{B}}_K \} + 1 \right)^{-1} \\ &\leq \left(\frac{\mathsf{snr}}{K} \lambda_K \{ \tilde{\mathbf{B}}_K^\dagger \mathbf{H}^\dagger \mathbf{H} \tilde{\mathbf{B}}_K \} + 1 \right)^{-1} \qquad (27) \end{aligned}$$

where $\lambda_k\{\cdot\}$ denotes the k^{th} ordered eigenvalue and for $k = 1, \ldots, K$ it holds $\lambda_k \{ \tilde{\mathbf{B}}_K^\dagger \mathbf{H}^\dagger \mathbf{H} \tilde{\mathbf{B}}_K \} \geq \lambda_K \{ \tilde{\mathbf{B}}_K^\dagger \mathbf{H}^\dagger \mathbf{H} \tilde{\mathbf{B}}_K \}$. Equivalently, we can lower-bound the instantaneous SNR as

$$\rho_K \geq \frac{\mathsf{snr}}{K} \lambda_K \{ \tilde{\mathbf{B}}_K^\dagger \mathbf{H}^\dagger \mathbf{H} \tilde{\mathbf{B}}_K \}. \qquad (28)$$

Substituting the SNR lower bound given in (28) back in the optimum selection function in (25), the optimum precoder $\tilde{\mathbf{B}}_{K,\mathrm{opt}}$ can be approximately selected from the codebook $\mathcal{B}_K$ as

$$\tilde{\mathbf{B}}_{K,\mathrm{opt}} \approx \arg \max_{\tilde{\mathbf{B}}_K \in \mathcal{B}_K} \frac{\beta_k}{K} \lambda_K \{ \tilde{\mathbf{B}}_K^\dagger \mathbf{H}^\dagger \mathbf{H} \tilde{\mathbf{B}}_K \}. \qquad (29)$$

The distortion function is then defined as the quadratic norm of the difference between the performance achieved with the optimum precoder with perfect CSI-T and with the best precoder in the codebook $\mathcal{B}_K$ using the approximate selection function in (29):

$$\begin{aligned} D(K, N_k) &= \frac{\beta_K}{K} \Big| \lambda_K \{ \mathbf{U}_K^\dagger \mathbf{H}^\dagger \mathbf{H} \mathbf{U}_K \} \\ &\quad - \max_{\tilde{\mathbf{B}}_K \in \mathcal{B}_K} \lambda_K \{ \tilde{\mathbf{B}}_K^\dagger \mathbf{H}^\dagger \mathbf{H} \tilde{\mathbf{B}}_K \} \Big|^2. \qquad (30) \end{aligned}$$

The codebook $\mathcal{B}_K$ is finally obtained as the set of N_K matrices that minimize the distortion in (30) averaged over all possible channel states and, thus, minimize the average performance loss of the limited feedback transceiver with respect to the perfect CSI-T case. It turns out that the distortion function in (30) coincides with the distortion function used in [10] and, hence, we can directly adopt the codebook design procedure proposed in [10] for Rayleigh fading channels. Since the codebook of [10] ensures full diversity (if $N_1 \geq n_T$), the proposed limited feedback multimode minimum BER linear transceiver fully exploits the diversity gain of the channel. Furthermore, we can also adopt the codebook allocation among the supported modes, i.e. the values of N_K given N, derived in [10] and this completes the codebook design problem.

Finally, once the codebooks $\{\mathcal{B}_K\}_{K \in \mathcal{K}}$ have been designed offline and are available at the transmitter and at the receiver, the limited feedback multimode linear transceiver works as follows: (i) the receiver estimates the channel and decides the optimum precoder

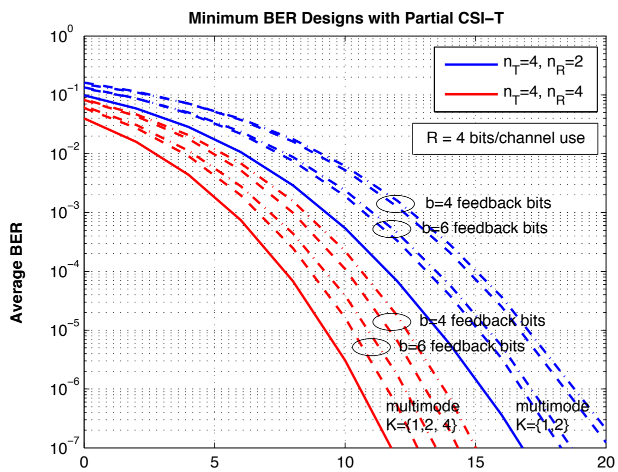

Fig. 3. Simulated average BER of the multimode minimum BER linear transceiver (solid), the limited feedback multimode minimum BER linear transceiver (dashed), and the limited feedback multimode precoder (dash-doted).

$\tilde{\mathbf{B}}_{K,\mathrm{opt}}$ using the selection function given in (25); (ii) the receiver feeds back the index corresponding to $\tilde{\mathbf{B}}_{K,\mathrm{opt}}$; (iii) the transmitter derives the optimum mode from the received index and constructs the optimum precoder by multiplying $\tilde{\mathbf{B}}_{K,\mathrm{opt}}$ with the rotation matrix $\mathbf{Q}_K$ as in (24).

4.3. Numerical Results

In Fig. 3 we provide the average BER performance of limited feedback multimode minimum BER linear transceiver (dashed) and of the suboptimum limited feedback multimode precoder of [10] (dash-dotted) in a Rayleigh flat-fading channel for $b = \{4, 6\}$ feedback bits. For comparison purposes, we also include the multimode minimum BER design (solid) when perfect CSI-T is available. Numerical simulations show that the proposed limited feedback minimum BER linear transceiver outperforms the scheme of [10]. In a practical situation this performance gap may imply reducing the feedback requirements in one bit under the same target quality of service. In any event, both schemes perform close to the system with perfect CSI-T with a small number of feedback bits, which, depending on the time-varying nature of the scenario, can be even negligible.

5. CONCLUSIONS

In this paper we have addressed the optimization of linear MIMO transceiver under a minimum BER criterion. In order to tackle the limitations arising from the common practice of a priori fixing the number of symbols to be transmitted, we have derived a multimode procedure that adaptively selects the optimum number of modes for a given target rate. The proposed multimode minimum BER design achieves the full diversity of the channel in both the perfect and partial CSI-T scenarios.

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7. REFERENCES

[1] I. E. Telatar, "Capacity of multi–antenna Gaussian channel," *European Trans. Telecomm.*, vol. 10, no. 6, pp. 585–595, Nov./Dec. 1999.

[2] A. Scaglione, G. B. Giannakis, and S. Barbarossa, "Redundant filterbank precoders and equalizers Part I: Unification and optimal designs," *IEEE Trans. Signal Processing*, vol. 47, no. 5, pp. 1988–2006, July 1999.

[3] H. Sampath, P. Stoica, and A. Paulraj, "Generalized linear precoder and decoder design for MIMO channels using the weighted MMSE criterion," *IEEE Trans. Commun.*, vol. 49, no. 12, pp. 2198–2206, December 2001.

[4] D. P. Palomar, J. M. Cioffi, and M. A. Lagunas, "Joint Tx-Rx beamforming design for multicarrier MIMO channels: a unified framework for convex optimization," *IEEE Trans. Signal Processing*, vol. 51, no. 9, pp. 2381–2401, September 2003.

[5] Y. Ding, T. N. Davidson, Z.-Q. Luo, and K. M. Wong, "Minimum BER block precoders for zero-forcing equalization," *IEEE Trans. Signal Processing*, vol. 51, no. 9, pp. 2410–2423, Sept. 2003.

[6] L. G. Ordóñez, D. P. Palomar, A. Pagès-Zamora, and J. R. Fonollosa, "High SNR analytical performance of spatial multiplexing MIMO systems with CSI," *submitted to IEEE Trans. Signal Processing*, 2006.

[7] D. P. Palomar, M. Bengtsson, and B. Ottersten, "Minimum BER linear transceivers for MIMO channels via primal decomposition," *IEEE Trans. Signal Process.*, vol. 53, no. 8, pp. 2866–2882, Aug. 2005.

[8] D. J. Love and R. W. Heath, Jr., "Limited feedback unitary precoding for orthogonal space-time block codes," *IEEE Trans. Signal Processing*, vol. 53, no. 1, pp. 64–73, Jan. 2005.

[9] D. J. Love and R. W. Heath, Jr., "Limited feedback unitary precoding for spatial multiplexing systems," *IEEE Trans. Inform. Theory*, vol. 51, no. 8, pp. 2967–2976, Aug. 2005.

[10] D. J. Love and R. W. Heath, Jr., "Multimode precoding for MIMO wireless systems," *IEEE Trans. Signal Processing*, vol. 53, no. 10, pp. 3674–3687, Oct. 2005.

[11] A. Scaglione, P. Stoica, S. Barbarossa, G. B. Giannakis, and H. Sampath, "Optimal designs for space-time linear precoders and decoders," *IEEE Trans. Signal Processing*, vol. 50, no. 5, pp. 1051–1064, May 2002.

[12] P. Stoica and G. Ganesan, "Maximum-SNR spatial-temporal formatting designs for MIMO channels," *IEEE Trans. Signal Process.*, vol. 50, no. 12, pp. 3036–3042, Dec. 2002.

[13] L. G. Ordóñez, D. P. Palomar, A. Pagès-Zamora, and J. R. Fonollosa, "Analytical BER performance in spatial multiplexing MIMO systems," *Proc. IEEE Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, 2005.

[14] T. K. Y. Lo, "Maximum ratio transmission," *IEEE Trans. Commun.*, vol. 47, no. 10, pp. 1458–1461, Oct. 1999.

[15] D. J. Love, R. W. Heath, Jr., W. Santipach, and M. L. Honig, "What is the value of limited feedback for MIMO channels?," *IEEE Commun. Magazine*, pp. 54–59, Oct. 2004.

[16] S. Zhou and B. Li, "BER criterion and codebook construction for finite-rate," *IEEE Trans. Signal Processing*, 2006.