# RESOURCE ALLOCATION IN MULTIHOP OFDMA BROADCAST NETWORKS

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**ABSTRACT**

This paper considers resource allocation strategies for multihop OFDMA broadcast networks consisting of a source, $K$ destinations (users), and $K$ dedicated decode-and-forward relays per hop. Network scenarios of practical interest have to face the availability of partial and imperfect channel state information at the transmitters (source and relays) due to fast time-varying channels, limited-rate feedback, and/or feedback delays. We address the limiting situation in which the transmitters have knowledge of the channel pathloss only and hence OFDMA is a means of implementing frequency-diversity. In this context, the computation of the optimal resource allocation maximizing a weighted sum of the users' effective rates is not straightforward since it gives rise to a non-convex optimization problem.

We analyze the two-hop case and propose a suboptimal, computationally efficient solution for which we are able to upper bound the gap from the optimum. Fortunately, the gap can be shown to be zero for some networks. For a general network with an arbitrary number of hops we propose a simplified extension of the two-hop solution.

## 1. INTRODUCTION

Orthogonal frequency division multiple-access (OFDMA) is an implementation-amenable, efficient technique to combat frequency selectivity of wireless channels while frequency-multiplexing users in a flexible and modular manner. For these and other reasons, it results appealing for upcoming wireless network standards (see [1] for example). In single-hop OFDMA broadcast networks, centralized perfect channel state information (CSI) of all the links, if available, can be used to allocate resources adaptively. Hence, bandwidth, power, and rate can be efficiently assigned to match the instantaneous network conditions [2].

Such perfect network state information is likely not to be available in a broadcast multihop scenario. One the one hand, the amount of processing required to take advantage of perfect CSI in all the links of all the hops can be formidable (the complexity has been shown to be NP hard even for a single-hop network [3]) and possibly non-affordable. On the other hand, for sufficiently fast time-varying channels, the necessary CSI update interval can happen to exceed the capacity of the limited-rate feedback channels of the network. Even when it does not occur, propagation and processing delays on the feedback channels may result in outdated, useless CSI at the transmitters. Therefore, it is of practical interest to study the network scenario of all transmitters having perfect knowledge of the channels' pathloss, a slow varying parameter, and being ignorant of the per-subcarrier fading states.

In this case, the IEEE 802.16 standard with PUSC (Partial Usage of Subcarriers Channelization) [1] considers uniform power allocation among groups of subcarriers far apart so as to suffer uncorrelated fading. Under this transmission strategy, which can be regarded as a frequency diversity (FD) technique, each link capacity is given by the ergodic (or average) capacity, which exclusively depends on the link signal-to-noise ratio and bandwidth for a given fading channel statistic, a sufficiently high number of carriers, and sufficiently frequency selective channels.

We address the optimization of the use of the network resources (power, bandwidth, and per-hop transmission time) for the maximization of a weighted sum of the effective rates of the users of a time division duplex (TDD) multihop OFDMA broadcast network with FD. A previous approach on time and bandwidth allocation for a multihop network is [4], where the multiuser case is not tackled. We formulate the computation of the optimal resource allocation as an optimization problem that turns out to be non-convex. Therefore, there is no efficient algorithm to compute the exact optimal allocation in practice. For this reason, we study the use of simpler, computationally amenable, alternative methods giving rise to accurate suboptimal allocations.

In particular, we propose a suboptimal solution specific for the two-hop network that can be numerically evaluated in practice (i.e., with a polynomial time worst-case complexity [5]) since it is based in the solution of simpler convex sub-problems. We are able to analytically determine the performance gap from the optimum and, fortunately, we find that it can be zero for some broadcast networks. For a general network with an arbitrary number of hops, we propose a simplified extension of the two-hop solution.

The rest of the paper is organized as follows. In Section 2 we address the system model and some preliminaries regarding approximations of the ergodic capacity. The computation of the global optimal allocation is formally stated in Section 3. Section 4 focuses on the proposal and analysis of a suboptimal solution for the two-hop broadcast network. The extension of a simplified version of the solution of Section 3 is proposed for the general network in Section 4. Section 5 analyzes the performance of the proposed solutions in some practical networks. Finally, Section 6 concludes the paper and sketches some lines for future work.

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## 2. SYSTEM MODEL AND PRELIMINARIES

### 2.1. System model

The broadcast multihop network model is shown in Figure 1. A source $\mathsf{S}$ of power $P_s$ sends independent information to $K$ destinations $\{\mathsf{D}_1, \ldots, \mathsf{D}_K\}$ through $N$ hops (1 broadcast hop plus $(N-1)$ relaying hops), in each of which there are $K$ dedicated relaying terminals $\{\mathsf{R}_1^n, \ldots, \mathsf{R}_K^n\}$ of power $P_r$. We denote by $|h_{k,n}|^2$ the channel pathloss of the $k$-th link at the $n$-th hop.

In the broadcast hop, the $k$-th information stream is transmitted at a rate $R_{k,1}$ consuming a fraction $\gamma_k$ of the source power and a *disjoint* fraction $\beta_{k,1}$ of the total network bandwidth $W$, $1 \le k \le K$, which is fully reutilized in each subsequent hop. In the $n$-th relaying hop, the $k$-th relay decodes its dedicated information flow and forwards it at a rate $R_{k,n}$ using its total power $P_r$ over another *disjoint* fraction of $W$, $\beta_{k,n}$, $1 \le k \le K$, $2 \le n \le N$. By disjoint bandwidth fraction we mean that at the $n$-th hop, the $k$-th information flow is transmitted using a number of far apart subcarriers such that a bandwidth $\beta_{k,n}W$ is occupied in exclusivity. Since all the relays are assumed half-duplex there is no interference at all in the system, and we denote by $\alpha_n$ the fraction of time devoted to the transmission of information in the $n$-th hop. Therefore, a resource allocation $\mathcal{F}$ for the network of Figure 1 consists of

$$\mathcal{F} = \{\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_N, \boldsymbol{\gamma}, \boldsymbol{\alpha}\} \in \mathcal{A}_K^{N+2}, \tag{1}$$

where $\boldsymbol{\beta}_n = [\beta_{1,n} \ldots \beta_{K,n}]^T$, $\boldsymbol{\gamma} = [\gamma_1 \ldots \gamma_N]^T$, $\boldsymbol{\alpha} = [\alpha_1 \ldots \alpha_N]^T$, and $\mathcal{A}_K \triangleq \{\mathbf{z} \in \mathbb{R}_+^K \mid \mathbf{1}^T\mathbf{z} = 1\}$.

### 2.2. Approximation of the ergodic capacity

Consider the situation where some power $P$ is uniformly allocated among a group of subcarriers experiencing uncorrelated fading and spanning a bandwidth $W$. At a given subcarrier, the attenuation is $|h|^2 f$, where $|h|^2$ is the channel pathloss and $f$ is the fading, and there is AWGN of one-sided PSD $N_0$. Without fading knowledge, the maximum transmission rate is given by the ergodic capacity

$$R \le W \mathrm{E}_f \left\{ \log_2\!\left(1 + \frac{|h|^2}{N_0 W} P f\right) \right\}, \tag{2}$$

which can be tightly lower bounded as

$$W \log_2\!\left(1 + \rho \frac{|h|^2}{N_0 W} P\right) \le W \mathrm{E}_f \left\{ \log_2\!\left(1 + \frac{|h|^2}{N_0 W} P f\right) \right\}, \tag{3}$$

where, for Rayleigh fading, $\rho \triangleq 2^{\mathrm{E}_f\{\log_2(f)\}} = e^{-\Psi}$ and $\Psi = 0.5772...$ is the Euler-Mascheroni constant (see [6, 4.352-1]). The lower bound follows from the application of Jensen's inequality to the function $f(x) = \log_2(1 + c2^x)$, $c > 0$. We can use the approximation (3) of the ergodic capacity to identify the maximum transmission rates of the $k$-th data flow in the broadcast hop

$$R_{k,1}(\beta_{k,1}, \gamma_k) \triangleq \beta_{k,1} W \log_2(1 + c_{k,1}\gamma_k/\beta_{k,1}) \tag{4}$$

and in any relaying hop $2 \le n \le N$,

$$R_{k,n}(\beta_{k,n}) \triangleq \beta_{k,n} W \log_2(1 + c_{k,n}/\beta_{k,n}), \tag{5}$$

where

$$c_{k,n} \triangleq \begin{cases} \rho \frac{|h_{k,1}|^2}{N_0 W} P_s & n = 1 \\ \rho \frac{|h_{k,n}|^2}{N_0 W} P_r & 2 \le n \le N \end{cases}. \tag{6}$$

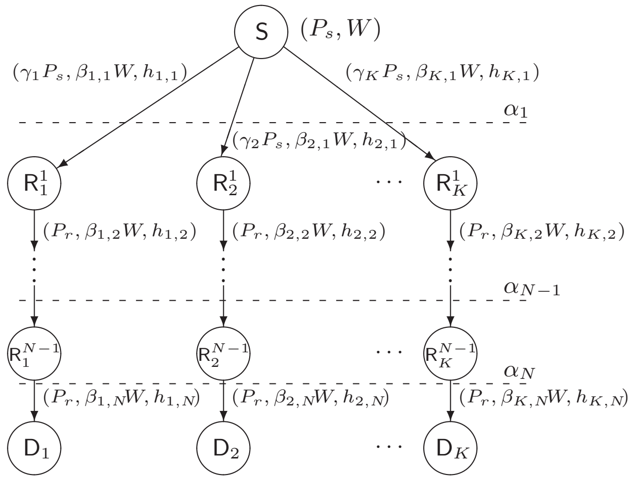


**Fig. 1**. A $K$-user $N$-hop FD multihop OFDMA broadcast network.

### 2.3. Per-hop achievable rate regions

We denote by $\mathcal{R}_n \subset \mathbb{R}_+^K$ the achievable rate region of the $n$-th hop, i.e.,

$$\mathcal{R}_1 \triangleq \bigcup_{\boldsymbol{\beta}_1, \boldsymbol{\gamma} \in \mathcal{A}_K^2} \{\mathbf{R} \in \mathbb{R}_+^K \mid R_k \le R_{k,1}(\beta_{k,1}, \gamma_k), 1 \le k \le K\}, \tag{7}$$

$$\mathcal{R}_n \triangleq \bigcup_{\boldsymbol{\beta}_n \in \mathcal{A}_K} \{\mathbf{R} \in \mathbb{R}_+^K \mid R_k \le R_{k,n}(\beta_{k,n}), 1 \le k \le K\} \tag{8}$$

where $2 \le n \le N$. Since $\{\mathcal{R}_n\}_{n=1}^N$ are convex sets, we can efficiently compute $\mathbf{x}_n(\boldsymbol{\mu}) = [x_{1,n}(\boldsymbol{\mu}) \ldots x_{K,n}(\boldsymbol{\mu})]^T$, the solution to

$$\underset{\mathbf{x}}{\text{maximize}} \quad \boldsymbol{\mu}^T \mathbf{x} \tag{9}$$

$$\text{subject to} \quad \mathbf{x} \in \mathcal{R}_n \tag{10}$$

for any $\boldsymbol{\mu} \in \mathbb{R}_+^K$, yielding an objective value $C_n(\boldsymbol{\mu}) = \boldsymbol{\mu}^T \mathbf{x}_n(\boldsymbol{\mu})$, the weighted sum rate in hop $n$. Expressions (7)-(10) will allow us to compact the formulation used in subsequent sections.

## 3. OPTIMAL RESOURCE ALLOCATION

Due to the *decode-and-forward* behavior of the relays, the effective rate of communication of one user may be affected by bottlenecks in the hops. Hence, if we denote by $R_k$ the effective rate[1] of user $k$,

$$R_k \le \min_{1 \le n \le N} \{\alpha_n R_{k,n}\}, \tag{11}$$

where the half-duplex nature of the relays comes into play with the time-sharing vector $\boldsymbol{\alpha}$. Clearly, the achievable rate vector $\mathbf{R} = [R_1 \ldots R_K]^T$ depends on the allocation $\mathcal{F}$ through the per-hop rates and the time-sharing. We fix as the optimization criterion the maximization of a weighted sum of the users' effective rates

$$\sum_{k=1}^{K} \mu_k R_k = \boldsymbol{\mu}^T \mathbf{R}, \tag{12}$$

which is of practical interest since

- $\boldsymbol{\mu}$ can be the output of a packet scheduler at the source, updated according to a quality of service criterion [7].
- varying $\boldsymbol{\mu}$ we obtain the achievable rate region of the network.

[1] Note that the dependence of $R_{k,1}$ on $(\beta_{k,1}, \gamma_k)$ and the dependence of $R_{k,n}$ on $\beta_{k,n}$ has been omitted in (11) for brevity.

### 3.1. Formulation

The computation of the optimal allocation can be formulated as the solution to the following optimization problem,

$$\underset{\{\mathcal{F},\mathbf{R}\}}{\text{maximize}} \quad \boldsymbol{\mu}^T\mathbf{R} \tag{13}$$

$$\text{subject to} \quad R_k \le \min_{1\le n\le N}\{\alpha_n R_{k,n}\},\ 1\le k\le K \tag{14}$$

$$\mathcal{F}\in\mathcal{A}_K^{N+2} \tag{15}$$

which is non-convex because of the constraints (14). Note that the RHSs of (14) are not concave (they should be concave for the problem to be convex) since they contain non-concavities of the form $zy\log_2(1+x/y)$ and $zy\log_2(1+1/y)$ (see equations (4) and (5)). By using (7)-(8), an equivalent formulation of (13)-(15) is

$$\underset{\{\boldsymbol{\alpha},\mathbf{R}\}}{\text{maximize}} \quad \boldsymbol{\mu}^T\mathbf{R} \tag{16}$$

$$\text{subject to} \quad \mathbf{R}\in\alpha_n\mathcal{R}_n, 1\le n\le N \tag{17}$$

$$\boldsymbol{\alpha}\in\mathcal{A}_K. \tag{18}$$

Let us denote by $(\boldsymbol{\alpha}^\star(\boldsymbol{\mu}),\mathbf{R}^\star(\boldsymbol{\mu}))$ the solution to (16)-(18).

### 3.2. Centralized exhaustive search computation

The non-convexity of the problem (13)-(15) is due to the optimization of $\boldsymbol{\alpha}$. To that end, consider expression (17), which is equivalent to

$$\mathbf{R}\in\bigcap_{n=1}^{N}(\alpha_n\mathcal{R}_n). \tag{19}$$

Since the intersection of convex sets is another convex set [5], for fixed $\boldsymbol{\alpha}$ the problem (16)-(18) is convex and it can be efficiently solved. Hence, we can tackle the joint optimization of (13)-(15) by performing an exhaustive search over $\boldsymbol{\alpha}$ where, for each vector under test, (16)-(18) is solved using an interior point method. We name this impractical solution *centralized* because it requires knowledge of *all* the channel pathlosses of the network to obtain the optimal resource allocation[2] (it requires knowledge of $\{\mathcal{R}_n\}_{n=1}^N$). Fortunately, this solution scales well with $K$, which only affects the convex part of the proposed algorithm.

In the next sections we propose alternative suboptimal methods to find the time-sharing weights $\boldsymbol{\alpha}$ in an efficient manner to avoid the exhaustive search. We focus first on the two-hop network, and later on the general network.

## 4. PROPOSED RESOURCE ALLOCATION FOR THE $N=2$ NETWORK

In a broadcast network with two hops, $N=2$, the computation of the optimal allocation simplifies to

$$\underset{\{\alpha,\mathbf{R}\}}{\text{maximize}} \quad \boldsymbol{\mu}^T\mathbf{R} \tag{20}$$

$$\text{subject to} \quad \mathbf{R}\in\alpha\mathcal{R}_1 \tag{21}$$

$$\mathbf{R}\in(1-\alpha)\mathcal{R}_2 \tag{22}$$

$$0\le\alpha\le1, \tag{23}$$

In this section we will exploit the geometrical structure of the non-convex optimization problem (20)-(23) to propose a method for obtaining a suboptimal resource allocation. The proposed resource allocation is shown to be optimal under some conditions, which are obtained from the following results.

[2]Note that once $\mathbf{R}^\star(\boldsymbol{\mu})$ and $\boldsymbol{\alpha}^\star(\boldsymbol{\mu})$ are obtained, the optimal allocation $\mathcal{F}^\star$ is uniquely determined.

**Lemma 1.** *The optimal time-sharing parameter $\alpha^\star(\boldsymbol{\mu})$ satisfies*

$$\mathbf{R}^\star(\boldsymbol{\mu})\in\mathrm{bd}(\alpha^\star(\boldsymbol{\mu})\mathcal{R}_1)\cap\mathrm{bd}((1-\alpha^\star(\boldsymbol{\mu}))\mathcal{R}_2), \tag{24}$$

*where $\mathrm{bd}(\cdot)$ is the boundary of a set.*

*Proof.* Suppose that $\mathbf{R}\in\mathrm{bd}((1-\alpha)\mathcal{R}_2)\cap\mathrm{int}(\alpha\mathcal{R}_1)$[3]. Hence, for an arbitrarily small $\epsilon>0$, there exists another feasible vector $\mathbf{R}'=(1+\epsilon/(1-\alpha))\mathbf{R}$ and $\alpha'=\alpha-\epsilon$ such that $\mathbf{R}'\in\mathrm{bd}((1-\alpha')\mathcal{R}_2)$ and $\mathbf{R}'\in\alpha'\mathcal{R}_1$ yielding an objective value $(1+\epsilon/(1-\alpha))$ times larger. Hence, $\mathbf{R}$ and $\alpha$ cannot be optimal. Similarly, an optimal pair $(\mathbf{R},\alpha)$ cannot satisfy $\mathbf{R}\in\mathrm{bd}(\alpha\mathcal{R}_1)\cap\mathrm{int}((1-\alpha)\mathcal{R}_2)$. □

Lemma 1 implies that a necessary condition for a rate vector $\mathbf{R}$ to be optimal is that some time-sharing parameter $\alpha$ exists such that the boundaries of $\alpha\mathcal{R}_1$ and $(1-\alpha)\mathcal{R}_2$ cross at $\mathbf{R}$. So far, we have not been able to prove sufficiency of this condition. Lemma 1 will inspire a more general resource allocation algorithm for networks with an arbitrary number of hops in Section 5.

**Lemma 2.** *The optimal allocation from* (20)-(23) *can be equivalently found solving the following non-convex optimization problem*

$$\underset{\{d,\mathbf{y}\}}{\text{maximize}} \quad \boldsymbol{\mu}^T\mathbf{y}/(1+d) \tag{25}$$

$$\text{subject to} \quad \mathbf{y}\in(d\mathcal{R}_1)\cap\mathcal{R}_2 \tag{26}$$

$$d\ge0. \tag{27}$$

*Proof.* It follows from applying the variable change $\mathbf{y}=\mathbf{R}/(1-\alpha)$ and $d=\alpha/(1-\alpha)$ in (20)-(23). □

The advantage of Lemma 2 is to rephrase (20)-(23) in such a way that only one of the per-hop achievable rate regions is weighted by one optimization variable, simplifying subsequent analysis. Thanks to Lemma 2, $(\mathbf{y}^\star(\boldsymbol{\mu}),d^\star(\boldsymbol{\mu}))$, the optimal solution to (25)-(27) can be properly transformed into

$$\mathbf{R}^\star(\boldsymbol{\mu}) = \mathbf{y}^\star(\boldsymbol{\mu})/(1+d^\star(\boldsymbol{\mu})) \tag{28}$$

$$\alpha^\star(\boldsymbol{\mu}) = d^\star(\boldsymbol{\mu})/(1+d^\star(\boldsymbol{\mu})). \tag{29}$$

**Lemma 3.** *The objective value of* (25)-(27) *is strictly increasing for $d\in(0,d_1(\boldsymbol{\mu})]$ and strictly decreasing for $d\in[d_2(\boldsymbol{\mu}),+\infty)$, where*

$$d_1(\boldsymbol{\mu}) \triangleq \{d\ge0 \mid d\mathbf{x}_1(\boldsymbol{\mu})\in\mathrm{bd}(\mathcal{R}_2)\} \tag{30}$$

$$d_2(\boldsymbol{\mu}) \triangleq \{d\ge0 \mid \mathbf{x}_2(\boldsymbol{\mu})\in\mathrm{bd}(d\mathcal{R}_1)\}, \tag{31}$$

*$d_2(\boldsymbol{\mu})\ge d_1(\boldsymbol{\mu})\ \forall\boldsymbol{\mu}$, and the vectors $\mathbf{x}_n(\boldsymbol{\mu})$ are defined in* (9)-(10).

*Proof.* If $d\in[0,d_1(\boldsymbol{\mu})]$, $\mathbf{y}^\star(\boldsymbol{\mu})=d\mathbf{x}_1(\boldsymbol{\mu})$ and the objective value, $C_1(\boldsymbol{\mu})d/(1+d)$, is increasing in $d$. For $d\in[d_2(\boldsymbol{\mu}),+\infty)$, $\mathbf{y}^\star(\boldsymbol{\mu})=\mathbf{x}_2(\boldsymbol{\mu})$ and the objective value, $C_2(\boldsymbol{\mu})/(1+d)$, decreases in $d$. A contradiction argument on the monotony of the objective value of (25)-(27) is sufficient to prove $d_2(\boldsymbol{\mu})\ge d_1(\boldsymbol{\mu})$. □

Lemma 3 provides us with a means of reducing the complexity of the exhaustive search method of Section 3.2. Instead of drawing samples from the interval $[0,1]$ for finding $\alpha^\star(\boldsymbol{\mu})$, it is sufficient to sample the shorter interval $[\alpha_1(\boldsymbol{\mu}),\alpha_2(\boldsymbol{\mu})]$, where

$$\alpha_k(\boldsymbol{\mu})=\frac{d_k(\boldsymbol{\mu})}{1+d_k(\boldsymbol{\mu})},\ k=1,2, \tag{32}$$

[3]We denote by $\mathrm{int}(\cdot)$ the *interior* of a set.

are the time-sharing parameters associated to $\{d_k(\boldsymbol{\mu})\}$ (29). Since $d_1(\boldsymbol{\mu})$ and $d_2(\boldsymbol{\mu})$ can be efficiently computed by solving the following convex optimization problems,

$$d_1(\boldsymbol{\mu}) = \max_{d\geq 0} \ d \quad \text{s.t.} \quad d\mathbf{x}_1(\boldsymbol{\mu}) \in \mathcal{R}_2 \qquad 1/d_2(\boldsymbol{\mu}) = \max_{d\geq 0} \ d \quad \text{s.t.} \quad d\mathbf{x}_2(\boldsymbol{\mu}) \in \mathcal{R}_1, \tag{33}$$

the computational load associated to the computation of $\alpha_1(\boldsymbol{\mu})$ and $\alpha_2(\boldsymbol{\mu})$ is negligible.

Instead of sampling the interval $[\alpha_1(\boldsymbol{\mu}), \alpha_2(\boldsymbol{\mu})]$, we propose to tackle the exhaustive search associated to (20)-(23) by selecting the best candidate rate vector out of the ones obtained with $\alpha_1(\boldsymbol{\mu})$ and $\alpha_2(\boldsymbol{\mu})$. Hence, according to Lemma 3, the resulting achievable rate vector is $\mathbf{R}_{\text{sub}}(\boldsymbol{\mu})$[4] is

$$\mathbf{R}_{\text{sub}}(\boldsymbol{\mu}) = \begin{cases} \frac{d_1(\boldsymbol{\mu})}{1+d_1(\boldsymbol{\mu})}\mathbf{x}_1(\boldsymbol{\mu}) & \frac{d_1(\boldsymbol{\mu})C_1(\boldsymbol{\mu})}{1+d_1(\boldsymbol{\mu})} \geq \frac{C_2(\boldsymbol{\mu})}{1+d_2(\boldsymbol{\mu})} \\ \frac{1}{1+d_2(\boldsymbol{\mu})}\mathbf{x}_2(\boldsymbol{\mu}) & \frac{d_1(\boldsymbol{\mu})C_1(\boldsymbol{\mu})}{1+d_1(\boldsymbol{\mu})} < \frac{C_2(\boldsymbol{\mu})}{1+d_2(\boldsymbol{\mu})} \end{cases}, \tag{34}$$

where we have used that when $d = d_1(\boldsymbol{\mu})$, the optimal $\mathbf{y}$ is $\mathbf{y}^\star(\boldsymbol{\mu}) = d_1(\boldsymbol{\mu})\mathbf{x}_1(\boldsymbol{\mu})$ and when $d = d_2(\boldsymbol{\mu})$ the optimal $\mathbf{y}$ is $\mathbf{y}^\star(\boldsymbol{\mu}) = \mathbf{x}_2(\boldsymbol{\mu})$.

**Lemma 4.** *The relative performance gap of the suboptimal allocation (34), defined as*

$$\Gamma(\boldsymbol{\mu}) \triangleq \frac{\boldsymbol{\mu}^T\mathbf{R}^\star(\boldsymbol{\mu})}{\boldsymbol{\mu}^T\mathbf{R}_{\text{sub}}(\boldsymbol{\mu})} - 1, \tag{35}$$

*is upper bounded by*

$$\Gamma(\boldsymbol{\mu}) \leq \min\Big\{\frac{C_2(\boldsymbol{\mu})}{d_1(\boldsymbol{\mu})C_1(\boldsymbol{\mu})}, \frac{1+d_2(\boldsymbol{\mu})}{1+d_1(\boldsymbol{\mu})}\Big\} - 1. \tag{36}$$

*Furthermore, any rate vector dominating*[5] $\mathbf{R}_{\text{out}}(\boldsymbol{\mu}) = \mathbf{x}_2(\boldsymbol{\mu})/(1+d_1(\boldsymbol{\mu}))$ *is non-achievable.*

*Proof.* Since (20)-(23) and (25)-(27) are equivalent problems, their optimal objective value is equal and can be upper-bounded as

$$\begin{aligned}\boldsymbol{\mu}^T\mathbf{R}^\star(\boldsymbol{\mu}) &= \max_{\{d\geq 0, \mathbf{R}\}} \frac{\boldsymbol{\mu}^T\mathbf{y}}{1+d} \ \text{s.t.} \ \mathbf{y} \in (d\mathcal{R}_1) \cap \mathcal{R}_2 \overset{(a)}{\leq} \max_{\{d\geq 0, \mathbf{R}\}} \frac{\boldsymbol{\mu}^T\mathbf{y}}{1+d_1(\boldsymbol{\mu})} \ \text{s.t.} \ \mathbf{y} \in (d\mathcal{R}_1) \cap \mathcal{R}_2 \\ &\overset{(b)}{=} \max_{\mathbf{R}} \frac{\boldsymbol{\mu}^T\mathbf{y}}{1+d_1(\boldsymbol{\mu})} \ \text{s.t.} \ \mathbf{y} \in \mathcal{R}_2 = \frac{\boldsymbol{\mu}^T\mathbf{x}_2(\boldsymbol{\mu})}{1+d_1(\boldsymbol{\mu})} = \frac{C_2(\boldsymbol{\mu})}{1+d_1(\boldsymbol{\mu})},\end{aligned} \tag{37}$$

where (a) follows from Lemma 3 and (b) follows from $d^\star = +\infty$, which yields $(d^\star\mathcal{R}_1) \cap \mathcal{R}_2 = \mathcal{R}_2$. Expression (36) is obtained after plugging (37) into (35) and noticing that

$$\boldsymbol{\mu}^T\mathbf{R}_{\text{sub}}(\boldsymbol{\mu}) = \max\Big\{\frac{d_1(\boldsymbol{\mu})C_1(\boldsymbol{\mu})}{1+d_1(\boldsymbol{\mu})}, \frac{C_2(\boldsymbol{\mu})}{1+d_2(\boldsymbol{\mu})}\Big\}. \tag{38}$$

Finally, $\mathbf{R}_{\text{out}}(\boldsymbol{\mu})$ achieves the upper bound (37) for $\alpha = \alpha_1(\boldsymbol{\mu})$. □

Lemma 4 implies that, for networks satisfying $d_2(\boldsymbol{\mu}) = d_1(\boldsymbol{\mu})$ or $C_2(\boldsymbol{\mu}) = d_1(\boldsymbol{\mu})C_1(\boldsymbol{\mu})$ the suboptimal allocation strategy induced by (34) is global *optimal*. Although we have not been able to characterize analytically the set of zero-gap networks, we will see in Section 6 that for networks with practical operating conditions and non-zero gap, the suboptimal solution (34) can still perform indistinguishably to the optimal allocation obtained with the exhaustive search.

[4] Again, $\mathbf{R}_{\text{sub}}$ induces a unique suboptimal resource allocation $\mathcal{F}_{\text{sub}}$.

[5] A rate vector $\mathbf{R}_{\text{a}}$ dominates $\mathbf{R}_{\text{b}}$ if $R_{\text{a},k} \geq R_{\text{b},k}$ $1 \leq k \leq K$, and at least one inequality is strict.

## 5. DISTRIBUTED SUBOPTIMAL SOLUTION FOR A GENERAL NETWORK

The optimal centralized solution of Section 3.2 may become impractical when the number of hops becomes large; not only the complexity of the exhaustive search on $\boldsymbol{\alpha}$ becomes prohibitive but also the network feedback load for the exchange of $\{c_{k,n}\}$. To avoid these handicaps, we propose an alternative distributed suboptimal solution whose building blocks are maximizations over the resource allocation parameters of one hop at a time. It is inspired by the following result.

**Lemma 5.** *The optimal time-sharing vector $\boldsymbol{\alpha}^\star(\boldsymbol{\mu})$ satisfies*

$$\mathbf{R}^\star(\boldsymbol{\mu}) \in \bigcap_{n=1}^{N} \text{bd}(\alpha_n^\star(\boldsymbol{\mu})\mathcal{R}_n). \tag{39}$$

*Proof.* The proof follows similar arguments to the ones used for proving Lemma 1, which is the particularization of (39) for $N = 2$. It is omitted for lack of space. □

Lemma 5 provides us with a necessary condition for optimality of $(\boldsymbol{\alpha}^\star(\boldsymbol{\mu}), \mathbf{R}^\star(\boldsymbol{\mu}))$: $\boldsymbol{\alpha}^\star(\boldsymbol{\mu})$ should be such that all the weighted per-hop achievable rate regions $\{\alpha_n^\star(\boldsymbol{\mu})\mathcal{R}_n\}_{n=1}^N$ should intersect in $\mathbf{R}^\star(\boldsymbol{\mu})$. We propose in Algorithm 1 a suboptimal resource allocation that satisfies Lemma 5.

The rationale of the algorithm is to obtain a solution for which all the regions intersect in some point along the maximizing direction of the last hop, defined by $\mathbf{x}_N(\boldsymbol{\mu})$, since it is the one which delivers information *directly* to the users. We back-propagate $\mathbf{x}_N(\boldsymbol{\mu})$ and adjust the time-sharing parameters (40)-(41) so as to get the best out of each hop.

The benefits of this solution are twofold. First, its building blocks are the convex optimization problems (9)-(10) and (40), which can be efficiently solved. Second, each of the optimizations (40) involves the parameters of only one hop, allowing for a *distributed* implementation of Algorithm 1, starting in the $N$-th hop, propagating backwards up to the source, and then propagating again forward to communicate $\alpha$. This requires less parameter exchanges than the centralized solution.

**Algorithm 1** Suboptimal resource allocation for a general network

1: **for** a given $\boldsymbol{\mu}$ **do**
2: Compute $\mathbf{x}_N(\boldsymbol{\mu})$ by solving (9)-(10).
3: **for** $n = 1 \ldots N-1$ **do**
4: Compute

$$d_n \triangleq \underset{d\geq 0}{\text{maximize}} \ d \quad \text{subject to} \quad d\mathbf{x}_N(\boldsymbol{\mu}) \in \mathcal{R}_n \tag{40}$$

5: **end for**
6: **end for**
7: The suboptimal allocation is $\boldsymbol{\alpha}_{\text{sub}}(\boldsymbol{\mu}) = [\alpha/d_1 \ \alpha/d_2 \ldots \alpha]^T$, $\mathbf{R}_{\text{sub}}(\boldsymbol{\mu})$, where

$$\mathbf{R}_{\text{sub}}(\boldsymbol{\mu}) = \alpha\mathbf{x}_N(\boldsymbol{\mu}) \equiv \Big(1 + \sum_{n=1}^{N-1} d_n\Big)^{-1}\mathbf{x}_N(\boldsymbol{\mu}). \tag{41}$$

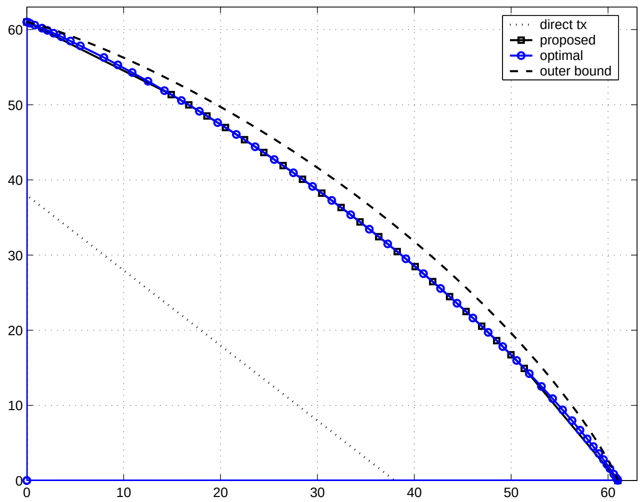


**Fig. 2**. Achievable rate regions in [Mbps] of the symmetric setting.

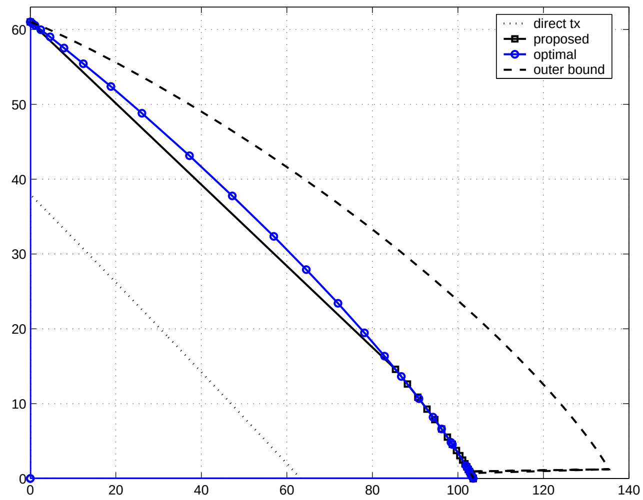


**Fig. 3**. Achievable rate regions in [Mbps] of the asymmetric setting.

## 6. PERFORMANCE EVALUATION

In this section, we compare the performance of the optimal (exhaustive search) and the distributed resource allocation (34) algorithms in a network with $K = 2$ users and $N = 2$ hops with realistic parameters for an 802.16 deployment. We simulate a network with $W = 20$ MHz, $P_s = 40$ dBmW, $P_r = 36$ dBmW, and $N_0 = -83.9$ dBmW/MHz. We assume an overhead of 25% pilot symbols for pathloss estimation which is taken into account on the effective network bandwidth and the achievable rates.

We consider two different network settings: one symmetric and another asymmetric. In the symmetric case, shown in Figure 2, the distance between the source and each of the relays is 240 m, while the distance between each relay and its intended destination is 160m. In the asymmetric case, shown in Figure 3, all the distances remain the same except the distance between relay one and user one, which now is only 50 m. In both cases, all the source-relay links are assumed in line-of-sight (LOS), with a pathloss exponent of 2.6. The relay-destination and source-destination links are assumed in non-LOS with an exponent of 4.05.

For both network settings we show the achievable rate region of (34) and an outer bound derived by plotting $\mathbf{R}_{\text{out}}(\boldsymbol{\mu})$ of Lemma 4 $\forall\boldsymbol{\mu}$. We use a computationally intensive exhaustive search method for the computation of the optimal allocation (20)-(23) as a benchmark. In particular, we optimize $\alpha$ by uniformly sampling the interval $[\alpha_1(\boldsymbol{\mu}), \alpha_2(\boldsymbol{\mu})]$ with 100 samples. Additionally, we provide comparison with a non-relayed network in which direct transmission from the source to the user terminals employs a total power $(P_s + 2P_r)$.

We can appreciate in Figures 2 and 3 significant rate gains derived from the use of the relayed network with respect to direct transmission. Furthermore, the proposed resource allocation method performs indistinguishably from the optimal allocation in the symmetric case, while in the asymmetric case it achieves most of the achievable rate region of the optimal allocation.

## 7. CONCLUSIONS

The maximization of a weighted sum of the users' effective rates in the context of a TDD OFDMA broadcast multihop network has been formulated as a non-convex optimization problem. Efficient methods for the *exact* computation of the optimal allocation do not exist and, instead, an impractical exhaustive search on the time-sharing fractions is required (fortunately, it is greatly alleviated thanks to Lemma 3 for the $N = 2$ case). We have derived an alternative computationally-efficient, suboptimal allocation algorithm based on the successive solution of convex optimization problems. Simulation results show that, for the two-hop case, the performance of the proposed solution is near-optimal in some practical scenarios although the performance gap is not zero. Future work will address the proof of sufficient conditions for optimality of the proposed allocation by, for instance, tightening the performance gap. Nonetheless, we will also focus on quantifying the performance gap of Algorithm 1 for a general network.

## 8. REFERENCES


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